Answer Sheet to the Written Exam

Corporate Finance and Incentives

December 2015

In order to achieve the maximal grade 12 for the course, the student must excel in all four problems.

The four problems jointly seek to test fulfillment of the course's learning outcomes: "Upon completion of the course, students will have developed an understanding of the different asset classes in financial markets as well as an approach on how to price them; including bonds, stocks, forwards, futures and options. Furthermore, the pricing methodology will also be used to illustrate how firms use these methods in order to choose their investment projects. Within the realms of Corporate Finance, we will explore the optimal capital structure of the firm, optimal dividend policy as well as how these and other factors influence how management runs the company."

Problem 1 is particularly focused on stock pricing methodology, problem 2 is particularly focused on the optimal capital structure, problem 3 is particularly focused on stocks, options and pricing models, while problem 4 finally has a broader coverage of the learning outcomes.

Some numerical calculations may differ slightly depending on the software used for computation, so a little slack is allowed when grading the answers.

Problem 1 (CAPM 25%)

1) To find the solution to $Az = b - r_f\mathbf{1}$, note that $z = A^{-1}(b - r_f\mathbf{1})$ and use matrix inversion in Excel. The solution for z is $(6, 3.75, -2.25, 7.5)^T$ which is normalized to the tangent portfolio $x_e = (0.4, 0.25, -0.15, 0.5)^T$.

2) The expected return is $x_e^T b = 0.14$ and the variance is $x_e^T A x_e = 0.00797$.

3) For firm i, the Beta is computed as $\beta_i^{eff} = Cov(R_i, R_{eff})/Var(R_{eff})$. The covariance is computed as $e_i^T A x_e$ where e_i is the unit vector for coordinate i. This provides that the covariances with the efficient portfolio are $(0.0064, 0.00885, 0.00785, 0.0075)$, and hence that the Betas are $(0.803, 1.110, 0.985, 1.098)$.

Problem 2 (Tax Shield 25%)

1) The value of the debt is $D = 2,000,000/2\% = 100$ million Kroner. Likewise, from present value 200 million Kroner, the annual pre-tax income is 4 million Kroner. After paying interest and taxes, every year the remaining amount of 1.3 million Kroner can go to equity owners, so $E = 65$ million Kroner. Every year, the interest payments reduce the tax payments by $700,000$ Kroner, so the present value of the interest tax shield is $700,000/2\% = 35$ million Kroner. The firm's levered post-tax value is then $D + E = 165$ million Kroner.

It might be noted that the unlevered firm would have paid 1.4 million Kroner in taxes every year, so the unlevered post-tax value is 130 million Kroner.

2) As long as the annual interest payment is maintained below the annual pre-tax income of 4 million Kroner, the cash-flow streams remain safe. Greater debt would reduce tax payments and increase the levered value of the firm. With a competitive issue of debt, it's the original shareholders who capture all gains from the arrangement.

3) Creditors obtain only half of the interest payments, so the market value of debt is reduced to half of what it was before, i.e., 50 million Kroner. Shareholders obtain 80% of what they got before, so the new equity value is 52 million Kroner. The firm's levered value is then 102 million Kroner.

If the firm had no debt, shareholders would obtain 80% of the firm's after-tax income of 2.6 million Kroner every year. This would be worth 104 million Kroner. So, the value of the tax shield is now negative at -2 million Kroner.

Issuing more debt will now benefit tax collectors at the expense of shareholders, opposite to part 2). The effective tax advantage of debt is negative.

Problem 3 (Binomial Model 25%)

1) At time 0, we need to compute the probability p such that

$$80 = \frac{p100.8 + (1-p)\,60.8}{1.01},$$

solved by $p = 50\%$. With the corresponding method, we find at time 1 at the higher node that the probability of the up-branch in the tree is is 75%. Finally, at time 1 at the lower node, the probability of the up-branch is 33.3%.

2) At time 2, the values from top to bottom are $(0, 28.19, 0, 18.592)$. At time 1 at the upper node, the value is

$$\frac{75\%0 + 25\%28.19}{1.01} = 6.978$$

and at the lower node the value is likewise 12.272. Finally, at time 0 the value is 9.530.

3) At time 2 the values are the same as in question 2). At time 1 at the upper node, the put option is out of the money, so there is no gain from exercising. Again, the value is 6.978 as computed before. At time 1 at the lower node, immediate exercising of the put option provides 9.2. This is less than the 12.272, so it is better not to exercise. Finally, at time 0, the put option is out of the money, so there is no gain from exercising. In sum, this American put option has the same value as the European option [which is not generally true for put options].

Problem 4 (Various Themes 25%)

1) The textbook argues this point in Section 10.6.

2) See the textbook's Section 30.4 or Section 3.5.1 in the notes by Lando and Poulsen.

3) See the textbook's Section 14.2.

4) See the textbook's Sections 16.1-3 for the definition, while Sections 17.5 and 30.1 discuss ways to reduce distress costs.